Transport in a Dissipative Luttinger Liquid

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We study theoretically the transport through a single impurity in a one-channel Luttinger liquid coupled to a dissipative (ohmic) bath . For non-zero dissipation η the weak link is always a relevant perturbation which suppresses transport strongly. At zero temperature the current voltage relation of the link is $I \sim \exp(-E_0/eV)$ where $E_0 \sim \eta/\kappa$ and κ denotes the compressibility. At non-zero temperature T the linear conductance is proportional to $\exp(-\sqrt{\mathcal{C}E_0/k_BT})$. The decay of Friedel oscillation saturates for distance larger than $L_\eta \sim 1/\eta$ from the impurity.

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Introduction. One dimensional interacting electron systems behave quite differently from their higher dimensional counterparts. Whereas in two and three dimensions interacting electrons form a Fermi fluid characterized by the same quantum numbers as free electrons [1], the excitations in one dimensional electron systems are of collective nature - essentially because the electrons cannot avoid each other. To create a single electron requires the excitation of an infinite number of these collective phonon-like excitations (plasmons). In this case electrons form a new state of matter: the Luttinger liquid (LL) [2, 3].

Renewed interest in Luttinger liquids arises from progress in manufacturing narrow quantum wires with a few or a single conducting channel. Examples are carbon nanotubes [4], polydiacetylen [5], quantum Hall edges [6] and semiconductor cleave edge quantum wires [7]. More recently LL have been realized in the core of a screw dislocation in *hcp* ^{4}He crystals [8].

A clean LL is a marginal supersolid exhibiting both a power law decay of superfluid and $2k_F$ density correlations. It is described by a single parameter K which is a measure of the short range interaction between the electrons, with $K > 1$ for attractive and $K < 1$ for repulsive interaction, respectively. In a contact free measurement a clean Luttinger liquid has a finite ac conductance $G(\omega) = Ke^2/h$, provided the system size L is smaller than $L_\omega = v/\omega$ [9, 10, 11], v denotes the plasmon velocity. In the opposite limit $L \gg v/\omega$ the system has a complex conductivity $\sigma(\omega) \approx 2vG/(-i\omega + \varepsilon)$, similar to a superconductor.

An isolated impurity leaves the linear zero temperature conductance of the LL unchanged, provided the interaction is attractive. This result is independent of the strength of the impurity. For repulsive interaction, on the other hand, the conductance vanishes at zero temperature as $G \sim V^{2/K-2}$, where V is the applied voltage [12, 13]. At non-zero temperatures T the conductance $G \sim T^{2/K-2}$ remains finite even for $V \to 0$.

In realistic systems electrons in a LL couple to other degrees of freedom, e.g. to phonons or to electrons in nearby gates. These degrees of freedom act as a dissipative bath [14]. In particular it has been shown recently that the coupling of a clean LL to gate electrons results in an ohmic dissipation of the electrons in the LL [15]. Dissipation introduces a further length scale $L_\eta \approx 1/(K\eta)$ where η denotes the dissipation strength. On scales $L > L_\eta$ plasmons become diffusive and displacement fluctuations are strongly suppressed restoring translational long range order (Wigner crystal). If $L_\omega \gg L_\eta$, i.e. $Kv\eta \gg \omega$, the conductivity $\sigma = 2e^2/(h\eta)$ is finite which is paralleled by diverging superfluid fluctuations.

The question arises what is the influence of dissipation on the transport properties of disordered LL? This question will be addressed in the present paper. In particular we study the influence of weak dissipation on the tunneling of electrons through a single impurity. It turns out that the dissipation strongly suppresses tunneling and hence changes the power law dependence of the conductance on V and T to an exponential one.

The model. We consider spinless electrons in a one dimensional wire which are coupled to a dissipative bath. In the center of the wire is a single impurity of arbitrary strength. Following Haldane [16] we rewrite the charge density $\rho(x)$ as

$$\rho = \pi^{-1}(k_F + \partial_x\varphi)[1 + \cos(2\varphi + 2k_Fx) + \ldots]. \qquad (1)$$

where $\varphi(x)$ denotes a bosonic (plasmon) displacement field. The Euclidean action of the system then reads [3]

$$\frac{S}{\hbar} = \frac{1}{2\pi K}\int_{-L/2}^{L/2}\int_{-\hbar/2k_BT}^{\hbar/2k_BT} dx d\tau \Bigg\{ \frac{1}{v}(\partial_\tau\varphi)^2 + v(\partial_x\varphi)^2 \qquad (2)$$

$$- W(\varphi)\delta(x) + \frac{\pi K\eta}{2}\int_{-\hbar/2k_BT}^{\hbar/2k_BT} \frac{d\tau'(\varphi(x,\tau) - \varphi(x,\tau')^2}{[\hbar\beta\sin(\pi(\tau-\tau')/\hbar\beta)]^2}\Bigg\}.$$

For $K \ll 1$ (2) describes also a charge or spin density wave, $K \to 0$ corresponds to the classical limit. $W(\varphi)$ is the impurity potential which is a periodic function of periodicity π. The last part of (2) describes ohmic dissipation [14]. It was derived from a pure Luttinger liquid

coupled electrostatically to a metallic gate in [15] where it was found that such a coupling is only relevant for $K < K_\eta = 1/2$. This fact can be taken into account by writing $\eta = \eta_0 \xi_\eta^{-1}$ where ξ_η denotes the Kosterlitz-Thouless correlation length diverging at the transition $K \to K_\eta - 0$ and η_0 is a dimensionless coupling constant. Since there may be other sources of dissipation which survive also for $K > 1/2$ in the following we will assume that $\eta > 0$.

Effective action. In treating the influence of the impurity we first consider the case of zero temperature. After Fourier transforming $\varphi(x,\tau) = \int_\omega \int_k \varphi_{k,\omega} e^{ikx+i\omega\tau}$ where $\int_\omega \equiv \int \frac{d\omega}{2\pi}$ etc. the harmonic action reads

$$\frac{S_0}{\hbar} = \frac{1}{2\pi K}\int_\omega \int_k \left\{\frac{1}{v}\omega^2 + vk^2 + K\eta|\omega|\right\}|\varphi_{k,\omega}|^2. \quad (3)$$

The last term in (3) describes the (weak) damping of the plasmons of complex frequency $\omega_P \approx v(k + iK\eta/2)$. The upper cut-off for momentum and frequency are Λ and ω_c , respectively. $\omega_c^{-1} = \min\{v\Lambda, E_{\rm diss}/\hbar\}$ where $E_{\rm diss}$ corresponds to the maximal energy which can be dissipated by the bath. For simplicity we will assume that they both are of the same order of magnitude. We integrate now in the standard procedure over the degrees of freedom outside of the impurity. The resulting effective impurity action now reads

$$\frac{S}{\hbar} \approx \frac{1}{\pi K}\int_\omega \left\{\omega^2 + Kv\eta|\omega|\right\}^{1/2}|\phi_\omega|^2 - \int d\tau W(\phi) \quad (4)$$

where $\phi_\omega = \int d\tau \phi(\tau) e^{-i\tau\omega}$ and $\phi(\tau) \equiv \varphi(x=0,\tau)$. As can be seen from (4) for low frequencies, $\omega < \omega_\eta = v/L_\eta$, the behavior of the system is controlled by the dissipation. For very large values of ω and sufficiently small dissipation, $\eta \ll \Lambda/K$, the effective action includes also a contribution $\sim \int_\omega \omega^2 |\phi_\omega|^2$ [3].

Weak impurity. Since $W(\phi)$ is a periodic function we can decompose it in a Fourier series $W(\phi) = \sum_{n=1}^\infty w_n \cos(2n\phi)$. For weak impurity strength we can calculate the renormalization of $W(\phi)$ perturbatively. This gives for the flow equations of w_n

$$\frac{dw_n}{dl} = \left(1 - \frac{n^2 K}{\sqrt{1+\eta K e^l/\Lambda}}\right) w_n. \quad (5)$$

Since the effective action depends in a non-analytic way on the frequency there is no renormalization of K and η.

For $\eta = 0$ we recover from (5) the result of Kane and Fisher [12] namely that the impurity is a relevant perturbation for repulsive interaction and irrelevant for attractive interaction. As soon as the dissipation is switched on the downward renormalization saturates at a length scale $\Lambda^{-1} e^{l^*} \approx L_\eta$. Thus the impurity is a relevant perturbation even for attractive interaction, provided the dissipation remains finite in this region [17].

Strong impurity. In the case of a strong impurity potential the main contribution to the partition function will come from configurations $\phi(\tau) = n\pi$ with n integer, interrupted by kinks which connect these pieces. These multi-kinks configurations are saddle points $\phi_s(\tau)$ of the effective action (4). For very large impurity strength kinks are narrow and hence are characterized by the high frequency limit of (4). A solution with n kinks and n antikinks can then be written as

$$\phi_s(\tau) \approx \pi \sum_{i=1}^{2n} \epsilon_i \Theta(\tau - \tau_i), \;\; \epsilon_i = \pm 1, \;\; \sum_{i=1}^{2n} \epsilon_i = 0 \quad (6)$$

where $\Theta(\tau)$ is the Heaviside step function and τ_i the kink positions. In frequency space the saddle point configuration can be written as

$$\phi_{s,\omega} \approx \frac{i\pi}{\omega}\sum_{i=1}^{2n} \epsilon_i e^{i\omega\tau_i}. \quad (7)$$

The partition function then reads

$$Z = \sum_{n=0}^\infty \sum_{\epsilon_j = \pm 1} \frac{t^{2n}}{(2n)!}\int d\tau_1 ... d\tau_{2n} e^{\frac{2}{K}\sum_{i<j}\epsilon_i\epsilon_j f(\tau_i - \tau_j)} \quad (8)$$

Here $t = e^{-S_{\rm kink}/\hbar}$ is the tunneling transparency, $S_{\rm kink} \sim \sqrt{w_1}$ denotes the action of an isolated kink and $f(\tau)$ is the kink interaction

$$f(\tau) = \int_0^{\omega_c} d\omega \frac{\sqrt{\omega^2 + vK\eta|\omega|}}{\omega^2}\left(1 - \cos(\omega\tau)\right) \quad (9)$$

Using a soft cut-off $\sim e^{-\omega/\omega_c}$ in (9) one obtains in the limit of $\omega_c \tau \gg 1$ as an interpolating expression (up to a constant)

$$f(\tau) \approx \ln \omega_c \tau + (2\pi v \eta K \tau)^{1/2}. \quad (10)$$

The logarithmic interaction prevails for small kink-antikink distances τ whereas for larger τ the interaction exhibits power law behavior.

A strong impurity can alternatively be considered as a weak link between the two half-wires $x < 0$ and $x > 0$, respectively. It is therefore convenient to rewrite the partition function (8) in a form in which t denotes the strength of the non-linear terms. This can be done in the standard way [18, 19] with the result $Z = \int \mathcal{D}\theta e^{-S_{\rm eff}\{\theta\}/\hbar}$

$$\frac{S_{\rm eff}}{\hbar} = \frac{1}{\pi}\int_\omega \frac{K\omega^2}{\sqrt{\omega^2 + v\eta K|\omega|}}|\theta_\omega|^2 - 2t\int d\tau \cos 2\theta(\tau).$$

We can now consider the renormalization of t which is given by

$$\frac{dt}{dl} = \left(1 - K^{-1}\sqrt{1 + \eta K e^l/\Lambda}\right) t. \quad (11)$$

Again $\eta \equiv 0$ reproduces the known result [12] that the weak link is renormalized to zero for $K < 1$. In the dissipative case the link is renormalized to zero for all K provided η remains finite. Thus from (5) and (11) we conclude that the impurity is always a relevant perturbation [17].

Transport. Next we calculate the current I through the impurity. The tunneling rate Γ and hence the current follows from [20]

$$\hbar\Gamma = 2(k_BT)^{-1}\mathrm{Im}\ln Z(V), \quad Z = Z_0 + iZ_1 \tag{12}$$

where $Z(V)$ denotes the partition function in the presence of an external voltage V. Z_0 includes the (stable) fluctuations of the ϕ-field close to a local minimum $\phi = n\pi$ whereas Z_1 includes the voltage driven (unstable) fluctuations which connect neighboring minima $\phi = n\pi \rightarrow (n+1)\pi$. Since the the strength of the impurity is renormalized to large values we can use the saddle point approximation employed in the previous paragraph for the calculation of Z. To obtain the saddle point action it is sufficient to consider a configuration consisting of a pair of kinks with distance τ

$$\frac{S(\tau)}{\hbar} = \frac{2}{K}f(\tau) + 2S_{\mathrm{kink}} - \frac{eV}{\hbar}\tau - \ln\tau. \tag{13}$$

Here we have taken into account that the voltage creates a term $-e\int d\tau\phi V/\pi$ in S [21]. The last term is of entropic origin and describes fluctuations in the kink distance. The saddle point follows then from

$$\frac{2}{K}f'(\tau) - \frac{eV}{\hbar} - \frac{1}{\tau} = 0. \tag{14}$$

This gives in for larger voltages for the saddle point $\tau_c \approx (2/K-1)\hbar/(eV)$ and hence for the current

$$I \sim e^{-S(\tau_c)} \sim t^2\left(eV/\hbar\omega_c\right)^{2/K-1}, \quad eV \ll \hbar\omega_c \tag{15}$$

i.e. we reproduce the result of Kane and Fisher [12] for the conductance $G \sim (eV/\hbar\omega_c)^{2/K-2}$. On the contrary, in the dissipative limit $v\eta K\tau \gg 1$ we get for the saddle point $\tau_c \approx 2\pi\eta v\hbar^2/(e^2V^2K)$ and hence for $eV\kappa \ll \eta$

$$I \approx Ae^{-S(\tau_c)/\hbar} = A(V)\,t^2\exp\Big\{-\frac{4\eta}{\kappa eV}\Big\}. \tag{16}$$

Here we introduced the compressibility $\kappa = K/(\pi v\hbar)$. In the case of non-interacting electrons κ coincides with the density of states [22]. As expected, the dissipation reduces the current with respect to the Kane-Fisher result strongly. This means that the tunneling density of states is suppressed more than a power law [23]. The cross-over between (16) and (15) happens at $eV/\hbar \approx \eta v$. An independent calculation using Fermi's "golden rule" gives for the pre-factor $A(V) \sim (\eta/\kappa(eV)^3)^{1/2}$ [24].

Next we consider finite temperatures. In this case the extension of the τ-axis is finite. Hence there will be a cross-over from (16) to a new behavior when the instanton hits the boundary, i.e. if $\tau_c \approx \hbar/k_BT$. In the dissipation free limit this results in a conductance $G \sim (k_BT/\hbar\omega_c)^{2/K-2}$. In the dissipative case this eV to k_BT dependence cross-over happens for $eV \approx \sqrt{2\eta k_BT/\kappa}$. For larger T the current is given by

$$I \approx B(T)\exp\left\{-\sqrt{\frac{\mathcal{C}\eta}{\kappa k_BT}}\right\}V, \quad \kappa k_BT \ll \mathcal{C}\eta. \tag{17}$$

The calculation does not allow a precise determination of the numerical constant $\mathcal{C}$ ($\mathcal{C} = 8$ from the present approach). The result resembles variable range hopping but results here from sequential tunneling through individual impurities. An independent calculation gives $B(T) \sim T^2\,(\kappa T/\eta)^{1/4}$ [24].

A simple way to reproduce (17) follows from integrating the flow equation for the transparency (11) and truncating the RG flow at the thermal de Broglie wave length $L_T = \hbar v/k_BT$ of the plasmons [3]. This gives an effective tunneling transparency

$$t_{\mathrm{eff}} \approx t_0\frac{\hbar v\Lambda}{k_BT}\exp\left\{-\sqrt{\frac{4\eta}{\pi k_BT\kappa}}\right\}. \tag{18}$$

The total current is then proportional to t_{eff}^2 which agrees with (17) apart from the value of $\mathcal{C}$.

Friedel oscillations. Finally we calculate the charge density oscillations close to the defect. In the absence of dissipation they decay $\langle\rho(x)\rangle \sim \cos(2k_Fx)|x|^{-\alpha}$ where $\alpha = K$ for $K < 1$ and $\alpha = 2K-1$ for $K > 1$ [25]. To determine the averaged charge density we write

$$\pi\langle\rho(x)\rangle - k_F = \langle\partial_x\varphi\rangle + k_F\cos(2k_Fx)e^{-2\langle\varphi^2(x)\rangle}. \tag{19}$$

The disorder average in the first term vanishes. To calculate the second term we remark that in the presence of dissipation the impurity is always relevant, its strength (5) growths under renormalization. To determine the large scale behavior of $\langle\rho(x)\rangle$ it is therefore justified to assume that φ is fixed at the defect. The fluctuation $\langle\varphi^2(x)\rangle$ is therefore identical to $\frac{1}{2}\langle(\varphi(x)-\varphi(-x))^2\rangle$. From this one finds for $x \ll L_\eta$ $\langle\rho(x)\rangle \sim \cos(2k_Fx)|x|^{-K}$, i.e. the power law decay well known from Luttinger liquids. However for $|x| \gg L_\eta$ the amplitude behaves like

$$\langle\rho(|x| \gg L_\eta)\rangle \approx \frac{\pi^{-1}k_F\cos(2k_Fx)}{(2\sqrt{y^2+y}+2y+1)^K}\left(1+\frac{KL_\eta}{|x|}\right), \tag{20}$$

with $y = \omega_c/(v\eta K) \equiv \Lambda L_\eta$.

Many impurities. So far we considered the influence of dissipation on the transport through a single impurity. In the case of many randomly distributed impurities with mean spacing a we expect no influence of dissipation as long as $a \ll L_\eta$. This agrees with our previous findings on Gaussian disorder [26] where weak dissipation has to be taken into account to guarantee energy conservation.

In this case one finds variable range hopping with only a weak dissipation dependence if one restricts oneself to exponential accuracy. For $L_\eta \ll a$ it is suggesting that if $L_T \ll a$ the different impurities are decoupled and one observes single impurity behavior.

Conclusions. We have shown that the inclusion of weak ohmic dissipation has a dramatic effect on the transport properties of a LL through a single impurity. The voltage and temperature dependence of the conductance is reduced from power laws in the dissipation free case to an exponential dependence, eqs. (16), (17).

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